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Challenges in Reservoir Forecasting[1]

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The combination of geostatistics-based numerical geological models and finite difference flow simulation has improved our ability to predict reservoir performance. The main contribution of geostatistical modeling has been more realistic representations of reservoir heterogeneity. Our understanding of the physics of fluid flow in porous media is reasonably captured by flow simulators in common usage. Notwithstanding the increasing application and success of geostatistics and flow simulation there remain many important challenges in reservoir forecasting. This application has alerted geoscientists and physicists that geostatistical/flow models in many respects, are, engineering approximations to the real spatial distribution and real flow processes. This paper reviews current research directions and presents some new ideas of where research could be focused to improve our ability to model geological features, model flow processes, and, ultimately, improve reservoir performance predictions.

KEY WORDS: geostatistics, flow simulation, streamtubes, scale up, gridding, accuracy and precision, modeling uncertainty.

INTRODUCTION

Research at the Stanford Center for Reservoir Forecasting (SCRF) is directed toward the construction of predictive reservoir models that, by construction, honor all available reservoir data. These models are intended to provide reliable predictions of future reservoir performance at all stages of the reservoir life cycle (from discovery to abandonment). Predictions of future performance are nonunique because of the unavoidable uncertainty in the detailed distribution of rock and fluid properties in the subsurface. At each time point, provided that we can perform the flow simulations, an ensemble of equally probable geostatistical realizations provides a model of uncertainty in future reservoir performance. Despite the recognition and acceptance of uncertainty, the presentation of uncertainty, the assessment of how good uncertainty estimates are, and optimal decision making in the presence of uncertainty remain nontrivial challenges in reservoir forecasting.

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The available data for reservoir forecastng includes conceptual geological models, seismic, core, well log, well test, and historical production data. Each source of data carries information, at different scales and with different levels of precision, related to the true distribution of petrophysical and fluid properties in the reservoir. One longstanding challenge is to integrate these disparate data into each geostatistical realization. Even as more sophisticated geostatistical tools are proposed, there becomes available more complex data, such as time-lapse seismic measurements and measurements from permanent downhole pressure gauges. Accounting for all of the available data *by construction* in numerical geological models will make it possible for reservoir engineers to perform quickly reservoir forecasting studies. Reducing the calendar and professional time for flow studies will add significant value to organizations managing natural resources.

From a reservoir engineering perspective, the critical petrophysical attribute is permeability. The spatial connectivity of high- and low-permeability features has an overwhelming influence on fluid flow, particularly for displacement processes. Features such as fractures and thin shale barriers may represent a small volumetric fraction of the reservoir and yet control the flow performance of the reservoir. For many geological environments, reproducing the *connectivity of extremes* remains an important challenge.

Geostatistical realizations are constructed at a fine scale to honor the greatest amount of data including features such as fractures and thin shales. The effects of these heterogeneities must be represented at a fine scale both for detailed small-area/cross-sectional flow simulation models as well as in more coarsely gridded full-field flow simulation models. Flow simulation must consider a relatively coarse representation of the reservoir compared to these fine-scale models. The translation of fine-scale geostatistical models to these more coarsely gridded flow models involves the two steps of gridding and scale up. Although progress has been made during the last two decades, there remain outstanding issues related to optimal gridding and scale-up.

Flow simulation solves well-established fluid flow equations based on conservation equations and auxiliary relations such as Darcy's law and its modifications for multiphase flow. A number of approximations are needed in the representation of pore-scale flow by effective properties of coarse flow simulation blocks. The challenge is to improve the quality of these approximations by increased understanding of the physics of fluid flow.

DATA INTEGRATION

The first step in reservoir forecasting is the construction of detailed lithofacies, porosity, and permeability models. There always will be a need to construct such stochastic reservoir models; although improved measurement tools such as high-resolution seismic data are of unquestioned importance, these advanced measurement techniques will not provide a deterministic map of the lithofacies, porosity, permeability, and saturation at the required scale. Geostatistical models reconcile the available data into a consistent numerical framework, estimate fluid volumes, quantify connectivity, and provide direct numerical input to flow simulation.

The challenge is to construct detailed rock property models that honor all of the available data. This task is made difficult because of the variety of data available. There is the problem of *scale*, for example, seismic data may represent a volume about one billion times larger than a core plug. A second problem is that of *precision*, for example, well-test derived properties are not precise as a result of model limitations. There also is the problem of *conceptual* data, for example, an interpretation of the regional geology may provide information on certain global features. Techniques in usage, such as Gaussian, indicator, or object-based methods are unable to address the variety of data available for reservoir modeling. For example, these techniques cannot account for the relatively large volume informed by 3D seismic attribute data.

Geologic Realism

Geostatistical models are meant to represent geological features in a way that captures their impact on reservoir performance; we are not interested in pretty pictures. We are interested in statistical/numerical descriptions of the important geologic features that have a first-order impact on fluid flow. As stated, the most important features are connected high and low permeability features. A challenge is to model such important features in the wide range of reservoir depositional settings encountered in practice. The continuity of geological features is almost always nonlinear which needs more advanced tools than two-point statistics such as the variogram. Another challenge is to develop more advanced measures of spatial connectivity that account for sound sedimentological/geological principles and forward models of such processes.

One promising area is to extend the hierarchical modeling scheme introduced in this issue of *Mathematical Geology* (Deutsch and Wang, 1996) where the local coordinate system is changed at each scale to conform to the dominant direction of continuity.

Seismic Data

Seismic data have long been used to map structural features such as major faults and geologic horizons. 3D seismic attribute data, however, have been used increasingly to assist in modeling the distribution of lithofacies and porosity. There are problems with scale and precision that must be overcome in such modeling; seismic data are at a coarser vertical resolution than geological and flow models and the properties measured by seismic data are related imprecisely to the petrophysical properties of interest (Lumley, 1995).

Repeated time-lapse 3D seismic surveys will become commonplace; the data brought by these types of measurements will provide more refined assessments of rock properties in the reservoir and, perhaps, a coarse-scale measurement of fluid movement. A challenge will be the integration of these data into fine-scale geostatistical models.

Dynamic Production Data

Dynamic production data consist of time-dependent pressure, flow rate, or tracer concentration measurements. From a reservoir engineering perspective, these data are perhaps the most important because they provide a direct measurement of the actual reservoir response. For reservoir prediction, a significant challenge is to relate these data to the underlying rock and fluid property distributions. It is essential to understand the physical phenomena in the reservoir that caused the dynamic production data to respond in a particular way. For example, dynamic data may

- provide general conceptual information on heterogeneities or specific information in areas of a reservoir,
- indicate the presence of reservoir boundaries (faults, lithofacies changes, stratigraphic pinchouts, etc.),
- provide information about the presence of heterogeneities such as sub-seismic fractures, and faults,
- measure the effective transmissibility of different regions in the reservoir,
- imply continuity between specific well locations,
- provide information about multiphase flow properties such as relative permeability and capillary pressure, or
- indicate a time-dependent change of reservoir properties related to geochemical alterations, pressure effects, or transport of solids within the resevoir.

In our opinion, it is extremely important to associate the correct physical interpretation to each aspect of the available production data and then to account for the data in a manner that (1) honors the physics of the measured data, (2) accounts for the volume scale of the data, (3) acknowledges and models the precision of the observation, and (4) permits a rigorous quantification of uncertainty. Intrepreting and understanding dynamic production data in this context is a significant outstanding challenge.

After understanding the data, the next step is to integrate the data into stochastic numerical geological reservoir models. There must be a link between the interpretation of the production data and the algorithm used, for example, object-based or discrete-event-type modeling techniques may be appropriate for geometrically clear lithofacies associations and faults. Simulated annealing or Markov Chain Monte Carlo methods already have shown some promise; however, a challenge is to demonstrate the successful implementation of such algorithms for practical problems. Accounting for production data in geostatistical reservoir modeling is a difficult inverse problem.

Continuity of Extremes

As mentioned, flow processes are particularly sensitive to the continuity of extreme low and high permeabilities. Large-scale features such as faults, stratigraphic pinchouts, and regional lithofacies changes are the most consequential. Smaller scale features such as subseismic faults, fractures, shales, and other stratigraphic features become important for scale-up and the assignment of effective flow properties.

Future work in this area will concentrate on (1) identifying and understanding connected permeability features in outcrops, (2) quantifying the connectivity with nonlinear connectivity measures such as multiple point statistics, object geometries, and pseudo-genetic-type simulation algorithms, and (3) building stochastic numerical models that integrate these statistical measures.

CHANGES OF SCALE

Whenever there are variations in permeability or relative permeability at scales which are smaller than the size of the flow simulation modeling cell, there will be a need for some scale averaging of the effective flow properties which are assigned to those cells. Similarly, whenever the rock volumes which inform a given measurement are large compared to the modeling cell, some downscaling of the integrated response measured by the data must be done to reproduce the variations at the scale of the modeling cell which gave rise to it.

Scaling up

The first problem, that of scale averaging, is a forward modeling problem, and consists of two parts. The first is referred to as the *flux problem*, that is the determination of an effective absolute permeability which will reproduce the total fluid flux observed in a fine-scale simulation in a given direction in response to an applied pressure gradient. The traditional approach has been to consider the group of fine-grid cells comprising a larger grid block in isolation from its neighbors and to apply a constant pressure difference to the opposing faces in one direction while imposing no-flow boundary conditions in the other. Because these are seldom the boundary conditions actually applied to that group of cells in their subsequent use in a field-scale reservoir model, the resulting permeabilities may be inappropriate for use in that model and produce the incorrect relationship between fluid flux and pressure drop.

When connected high or low values of the permeability are aligned in a direction which is neither parallel nor perpendicular to the direction of the pressure gradient, this cross-bedding can give rise to flow components which are orthogonal to the direction of the pressure gradient. The representation of this cross-flow requires the use of a full permeability tensor. The determination of all of the components of a permeability tensor requires the response of the group of cells to at least two sets of boundary conditions. Several methods for deriving permeability tensors have been proposed. These include: the use of more than two sets of boundary conditions and the determination of the tensor components from this overconstrained problem by linear regression (White and Horne, 1987); the use of just two boundary conditions applied to the group of cells being averaged along with a "skin" of neighboring cells surrounding them to place at some distance the influence of the imposed constant pressure and no-flow boundary conditions (Gomez-Hernandez and Journel, 1990); the use of the boundary conditions anticipated in large-scale simulation along with a perturbation of these to get the required pair of flow results (Pickup and others, 1992); and the use of two sets of periodic boundary conditions (Durlofsky, 1991). Of these proposed approaches, the use of periodic boundary conditions seems to be the most robust to the changes in boundary conditions which will be imposed on the coarse cell in subsequent large-scale simulation (Pickup and others, 1994).

However, even the permeability tensors derived with these sophisticated methods applied to groups of cells considered in isolation will fail to give the correct pressure-flux relations when applied to many problems of reservoir engineering interest. An example looked at by Yamada (1995) is that of a horizontal well producing in an anisotropic distribution of shales embedded in clean sands. The interaction of what would be a radial flow near the well in a homogeneous medium with the anisotropic shales led to errors in the pressure drawdown-flowrate relations of more than 100%, even for the tensors derived using periodic boundary conditions. Yamada (1995) derived a method for redefining the coarse cell boundaries to conform approximately to the streamlines and isobars for single-phase flow subject to the same boundary conditions, but the method requires a mapping of the flow response to a transformed space with the flow potential and streamfunction as coordinate axes, and a determination of the allowable cell connections in that space. Although he was successful in reproducing correctly the pressure-flux relations on his deformed grids for that particular problem, the method is difficult to generalize for arbitrary boundary conditions, and for all practical purposes, is restricted to flows in two dimensions. In addition, the appropriate definition of the deformed grids is specific to one set of boundary conditions, and requires redefinition if the boundary conditions are changed. The development of fast, robust, and straightforward procedures for scale averaging absolute permeability to preserve on coarse grids the pressure–flux relations observed in fine-scale simulations remains one of the major challenges in reservoir forecasting.

The second part of the scale averaging problem is referred to as the *transport problem*, that is the determination of the average effects of displacing fluid fronts moving through groups of fine-grid cells comprising a larger coarse-grid block. In the determination of absolute permeability, the influence of a thin streak of connected high permeability values aligned with the direction of flow may be averaged away when the pressure-flux response of the entire coarse-grid block is evaluated. However, in fluid displacement processes, these "channels" of high permeability present to the displacing fluid a preferential pathway which can have a significant impact on the breakthrough time and subsequent production of the displacing fluid from the coarse-grid block.

In miscible displacement processes in correlated, heterogeneous permeability fields, this effect leads to averaged displacement fronts which exhibit a nearly linear spreading behavior that cannot be described by the convection-dispersion equation describing miscible fluid displacements in homogeneous media (Hewett and Behrens, 1990). Reproduction of the effects of a linearly spreading displacement front requires the use of heuristic fractional flow theories (Koval, 1963; Todd and Longstaff, 1972; Fayers, 1988). These are not based on any fundamental understanding of the physics of miscible displacements in heterogeneous media, but can reproduce their displacement characteristics when calibrated for particular mobility ratios and heterogeneity patterns. Given the complexity of the interactions of unstable miscible displacement processes in correlated heterogeneity, including the effects of gravity, it is unlikely that a theory to describe them will ever be derived from first principles, but proper representation of these processes, nevertheless, remains a challenge in reservoir forecasting.

In immiscible displacement processes, it is important to recognize the different flow regimes which are characteristic of different length scales. At the smallest scales (mm to cm), the nature of displacements is controlled by the interplay of viscous and capillary forces, and the geometry of boundaries between permeability contrasts plays a critical role resulting from the abrupt changes in capillary pressure relations which are associated with permeability contrasts (Corbett and others, 1992). It is also at this scale that sedimentological features, such as cross-bedded laminae, are most regular and may be approximated as repetitive and nearly deterministic (Jensen and others, 1994), because their geometry is determined solely by the physics of the sedimentation processes which formed them. At this scale, it is possible to define effective relative permeability functions which reproduce the displacement behavior observed in fine-scale simulations and are also transportable to rocks in other locations with similar bedding styles.

At larger scales, the nature of displacements is controlled by the interplay of viscous and gravity forces with the correlated, but disordered, distributions of permeability. The distributions at this scale are controlled largely by the historical inputs to the sedimentation processes which formed them, that is the chaotic sequences of storms and weather cycles which drive sediment transport. There is a direct link between chaotic weather phenomena and random fractal processes hence the interest in fractal modeling techniques (Hewett, 1993). At this scale, the distributions are best reproduced using stochastic methods with relative permeabilities already calculated to account for the capillary-dominated phenomena as described.

In immiscible displacement processes, the effects of correlated heterogeneity have a similar effect to those in miscible displacement, leading to the early breakthrough of the displacing fluid, and modified fluid cuts following breakthrough. Fortunately, the modifications of Darcy's law to account for the interaction of immiscible fluids in homogeneous porous media already produce results which have a linear scaling behavior, through solutions of the Buckley-Leverett equation. Thus the effects of subgrid scale heterogeneity can be accounted for by modifying the relative permeabilities for the immiscible fluid phases, leaving the basic conservation equation unchanged. Unfortunately, although effective relative permeabilities derived from fine-scale simulations can reproduce the behavior of the fine-scale simulations from which they were derived, the modifications to the rock relative permeabilities measured at the core scale in the laboratory are specific to the particular distributions of permeability and relative permeability in the fine-scale model and the boundary conditions applied to them, so they are not transportable to other flow situations.

One of the major ambiguities in the definition of effective relative permeabilities for coarse grids is the need to decompose the coarse-grid block average of the product of absolute permeability, total fluid mobility ($k_{ro}/\mu_o + k_{rw}/\mu_w$), and potential gradient into separate averages for each quantity. Under the restrictive assumptions of vertical equilibrium, the potential gradient can be factored from the averages and the decomposition of the remaining product is unambiguous when the permeability is left unchanged from its single phase scale averaging result (Coats, Dempsey, and Henderson, 1971). When the conditions for vertical equilibrium do not pertain, the local potential gradient is coupled to the local permeability and total fluid mobility distributions, and this average product is not truly factorable. A number of alternative ways of defining the individual averages are possible, and nearly every possibility has been proposed in the literature (Fayers and Hewett, 1992). These include the use of a mobilityweighted average pseudo-potential in the definition of total mobility (Kyte and Berry, 1975), the use of an average of mobility values at a single plane to represent the average for the entire block (Stone, 1991), the use of a vertical arithmetic average pressure at grid block centers to calculate the average total mobility (Hewett and Behrens, 1990), and the use of a permeability-weighted average of the same pressures to calculate the average total mobility (Beier, 1992).

Recently, Hewett and Yamada (1995) proposed that the definition of effective relative permeabilities be based on the calculation of the integrated total mobility in streamtube segments extending between isobars through neighboring coarse-grid block centers. Because the integrated total mobility for each streamtube segment now is based on the same potential difference, it can be factored from the average of the streamtube integrated total mobilities and the definition of effective relative permeabilities is arrived at unambiguously. The resulting expression for the average integrated total mobility between coarse-grid block centers is

$$\overline{\lambda}_{t,n}(t) = \sum_{j=1}^{N_\psi} \left(\frac{T_n^j}{T_n} \right) \lambda_{t,n}^j(t)$$

which is a transmissibility-weighted average of the streamtube segment integrated total mobilities determined directly from the fine-scale fluxes between coarse-grid blocks and the potential difference between the coarse-grid block center isobars. Here, T_n^j and $\lambda_{t,n}^j$ are the single-phase transmissibility and integrated total mobility, respectively, of the individual streamtube segments. Using the relation for effective fractional flow common to all of the proposed methods, namely,

$$\overline{f}_{w,n}(t) = \sum_{j=1}^{N_\psi} \left(\frac{q_t^j(t)}{\overline{q}_t(t)} \right) f_{w,n}^j(t)$$

the relative permeabilities may be calculated as

$$\overline{k}_{rw}(t) = \overline{f}_w(t)\overline{\lambda}_t(t)\mu_w$$

$$\overline{k}_{ro}(t) = [1 - \overline{f}_w(t)]\overline{\lambda}_t(t)\mu_o$$

Combined with coincident values of $\overline{S}_w(t)$, the porosity-weighted average of the upstream grid block, we obtain $\overline{k}_{ro}(\overline{S}_w)$ and $\overline{k}_{rw}(\overline{S}_w)$, with the numerical dispersion correction introduced by Kyte and Berry (1975) because of the association of the fractional flow and total mobility for flow across the downstream face of the coarse-grid block with the average saturation of the upstream coarse-grid block. Although these results are exact for the definition of effective relative permeabilities proposed, they do not account for the effects of gravity. Generalization to include the effects of gravity will require separate consideration of the different phase potentials and the introduction of an effective capillary pressure at the coarse-grid block scale.

The uniqueness of the calculation of effective relative permeabilities to the particular distributions of absolute permeability, relative permeabilities, and the boundary conditions applied to them has been recognized since their introduction (Coats and others, 1967). Thus, they are not transportable to other flow situations and must be calculated separately for different portions of a reservoir and must be updated when boundary conditions change. As a practical matter, this requires that rapid method for evaluating them must be available. Recently, methods based on streamline time-of-flight calculations (Thiele, Blunt, and Orr, 1995) and semianalytical streamtube volumetric displacement calculations (Hewett and Yamada, 1995) have shown that accurate approximate predictions of oil-recovery performance can be obtained by mapping analytical one-dimensional solutions of the conservation equations onto the streamlines, or streamtubes, obtained from a steady-state, single-phase solution. These methods also can be applied to the calculation of effective relative permeabilities without the need for ever doing an unsteady, two-phase numerical simulation (Hewett and Yamada, 1995). Implementation of these ideas in a simulator where the effective flow properties are updated dynamically to reflect the changes in boundary conditions which are typical in any full-field reservoir simulation has yet to be demonstrated. This remains as an outstanding challenge in reservoir forecasting.

Scaling down

The second problem in scale averaging, that of downscaling the integrated response measured by production and seismic data for conditioning fine-scale representations of reservoir property distributions, is an inverse problem. All methods for the interpretation of seismic data rely on inverse methods. Methods for deriving the large-scale transmissivity distribution of an aquifer through an inverse problem conditioned on measured heads have been presented in the hydrology literature (de Marsily, 1982). Methods for automatic history matching of production data in field-scale reservoir simulations also have been presented (Fasanino, Molinara, and de Marsily, 1986). To date, however, these methods have not been able to address the requirements of fine-scale geostatistical modeling. Extensions of the theory of inverse modeling to allow for the incorporation of this type of data into the construction of fine-scale geostatistical reservoir models remains as another important challenge in reservoir forecasting.

GRIDDING FOR FLOW SIMULATION

Almost always, a coarse representation of the fine-scale numerical geological model is considered for flow simulation. The focus of the preceding discussion has been on the scale-up to effective flow properties; however, the geometry of the *coarse* flow simulation blocks must be defined before scale-up, that is there is a need for gridding prior to flow simulation. Considerations for gridding include (1) the grid should be aligned with major permeability features, (2) the grid blocks should have orthogonal boundaries for precision in finite difference flow simulators, (3) sufficient resolution is required to capture important flow phenomena such as gravity segregation and to ensure that numerical dispersion does not dominate the flow predictions, (4) greater resolution is required near wells where the flow rate is high, (5) for numerical precision, volumes of adjacent grid blocks should not be too different, and (6) the grid topology should lead to easy-to-solve and stable matrices in finite difference flow simulation.

One challenge is to optimize the grid size and geometry to get the best predictions possible for given computer resources. Engineers attempt this by performing small-scale gridding studies and applying guidelines established by past experience. A challenge is to employ powerful optimization techniques such as simulated annealing and genetic algorithms to get a more optimal solution and to automate the gridding procedure.

Voronoi grids have been developed that have the desirable feature that the block boundary between any two blocks is perpendicular to the line joining the block centers. This minimizes numerical errors in the solution to the flow equations. The challenge is to define representative transmissibilities in this setting where many of the block boundaries are not aligned with anisotropic permeability features.

Another attractive idea in gridding for flow simulation is to change the grid dynamically as the flow simulation proceeds. This would allow the grid to stay refined near fluid contacts and active wells even as these contacts and wells change with time. There are significant challenges managing numerical stability with this changing grid and the changing effective property distribution.

UNCERTAINTY QUANTIFICATION/DECISION-MAKING

There is uncertainty in the spatial distribution of rock and fluid properties, errors in the effective properties assigned during scale-up, uncertainty in multiphase flow properties, and many other approximations in flow simulation. These considerations lead to unquestionable uncertainty in our predictions of reservoir performance. We can say convincingly that the reservoir will perform the way the reservoir performs, that is there is no real uncertainty, it is the result of our ignorance and inability to model the physics of fluid flow with sufficient accuracy. One major challenge facing reservoir engineers and geoscientists is to quantify and minimize the unavoidable uncertainty in reservoir forecasting.

An increasing amount of reservoir-specific data becomes available from discovery to abandonment. The greatest challenge is to quantify uncertainty early in the reservoir life cycle when there is little data and when decisions have the most economic significance. Two other challenges are to establish (1) what data from other more mature fields should be borrowed, and (2) how to integrate these conceptual or global data into the local reservoir model.

A significant challenge is to assess the *goodness* of the geostatistical reservoir models. The two concepts of accuracy and precision could be considered for this purpose. A probability distribution is said to be accurate if the 10% symmetric probability interval (PI) contains the true value 10% (or more) of the time, the 20% PI contains the true value 20% (or more) of the time, and so on for increasingly wide probability intervals. To directly assess this measure of accuracy we require the true value for multiple probability distributions. In practice, this is possible with either the "leave-one-out" cross-validation approach or the "keep-some-back" jackknife approach. The idea is to build probabilistic distributions of uncertainty at multiple locations where the true values are known. Accuracy then may be judged by counting the number of times the true values actually fall within fixed probability intervals. Accuracy could be quantified for different probabilistic models (Gaussian, Indicator, Object-based, and Iterative/Annealing) and different implementation options. Precision is a measure of the narrowness of the distribution. Precision is defined only for accurate probability distributions; without accuracy, a constant value would have the ultimate precision. A probability distribution where the 90% PI contains the true value 99% of the time is accurate but not precise. Optimal precision is when the 90% PI contains the true value exactly 90% of the time. Systematically measuring the goodness of our stochastic reservoir models is a challenge that must be addressed.

Another challenge in reservoir forecasting is making optimal decisions in the presence of uncertainty. Decisions may be discrete: One platform or two? Horizontal or vertical wells? and so on. A distribution of uncertainty is useful because it tells us how much remains unknown; however, in many situations, a single *optimal* estimate or value is needed for decision-making. There are well-established methods in decision analysis, such as loss functions, for merging our assessments of uncertainty with the economic impact of making a mistake. Although these principles and some simple examples have been documented (Srivastava, 1990), the challenge of practical implementation remains.

CONCLUSIONS

The roots of most recent developments in reservoir characterization may be traced to other disciplines such as statistics, mathematics, numerical analysis, and physics. There remain many interesting theoretical and numerical developments in those other fields that have yet to be transferred to practical application in reservoir characterization and management. Some of these areas include inverse theory, optimization methods such as neural networks, mathematical areas of study such as graph theory, and percolation theory from physics. Although the concepts and theory exist, there are significant challenges involved with taking the hint of a good idea from basic science and developing it into a practical implementation of stochastic reservoir modeling.

This paper has given an overview of some challenges in reservoir characterization. The safe, environmentally sound, and economically responsible management of hydrocarbon reservoirs calls for a modeling of the spatial distribution of rock and fluid properties and subsequent flow modeling. Some of the challenges listed (e.g., scale up of multiphase flow properties) are specific to petroleum engineering. Many challenges, however, such as data integration and connectivity of extremes have wide applicability in many earth-science disciplines.

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